$$= -\frac{(a+ex_1)[a^2y_1y_1+b^2x_1x_2-a^2b^2)]}{a^2(a+ex_1)[x_1y_2-x_2y_1-ae(y_1-y_2)]} = -\frac{a^2y_1y_2+b^2x_1x_2-a^2b^2}{a^2[x_1y_2-x_2y_1-ae(y_1-y_2)]}.$$

It is seen that this expression is symmetrical with reference to x_1 and x_2, y_1 and y_2 with the exception of the sign, but considering that by finding tanPFB the slope of BF comes first, it is at once seen that tanPFB is the same as tanPFA. The difficulty of this method lies in the complicated algebraic work, which is avoided by using polar coördinates.

Solution of 255 by Prof. William Hoover was received after the solution in last issue had gone to press. Also a solution of 256 was received from a contributor who failed to sign his name.

NOTE. Professor Matz sent in a solution of 254 in which he points out that the line $x-4a=0$ is both tangent and normal to the curve. But the solution is not general. Who can give a general solution?

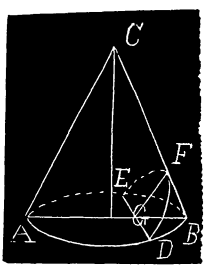
CALCULUS.

195. Proposed by CHRISTIAN HORNUNG, Heidelberg University, Tiffin, O.

Given a right cone of altitude h and radius r, to locate the plane parallel to its side which bisects the cone.

Solution by A. H. HOLMES, Brunswick, Maine, and J. SCHEFFER, Hagerstown, Md.

Let, in the right cone CAB, DEF represent a parabolic section. Put $BG=x$, $GE=y$, $FG=z$. The area of $DEF=\frac{4}{3}yz$; and consequently the volume of



$$BDEF=\tfrac{4}{3}\cdot\frac{h}{\sqrt{(r^2+h^2)}}\int_0^a yz\,dx,$$

and since $y^2=2rx-x^2$, and $z=\frac{x}{2r}\sqrt{(r^2+h^2)}$, we have for the volume, the integral

$$\tfrac{2}{3}\frac{h}{r}\int_0^x x\,dx\sqrt{(2rx-x^2)}=\tfrac{2}{3}\frac{h}{r}\left[\tfrac{1}{2}r^3\cos^{-1}\frac{r-x}{r}-\frac{3r^2+rx-2x^2}{6}\sqrt{(2rx-x^2)}\right].$$

To determine x for the condition that this volume is to be half the cone, we have the transcendental equation

$$2r^3\cos^{-1}\frac{r-x}{r}-\tfrac{2}{3}(3r^2+rx-2x^2)\sqrt{(2rx-x^2)}=r^3.$$

An approximate value of x is $x=1.3r$.

Also solved by R. D. Carmichael.

196. Proposed by F. P. MATZ, Sc. D., Ph. D., Reading, Pa.

The shortest tangent intercepted by the axes of the ellipse to which the tangent is drawn, equals the sum of the semi-axes of the ellipse.

I. Solution by the PROPOSER.

Tangent$=y\sqrt{[1+(dx/dy)^2]}+x\sqrt{[1+(dy/dx)^2]}$, in which $y=(b/a)\times\sqrt{(a^2-x^2)}$ and $dy/dx=-bx/a\sqrt{(a^2-x^2)}$.

$$\therefore U=\frac{1}{a}\left[\frac{\sqrt{(a^2-x^2)}}{x}+\frac{x}{\sqrt{(a^2-x^2)}}\right]\sqrt{[a^4-(a^2-b^2)x^2]}$$

$$=a\sqrt{\left(\frac{a^4-(a^2-b^2)x^2}{x^2(a^2-x^2)}\right)}=\text{a minimum.}$$

$\therefore x=a^3/(a+b)$; and, consequently, the length of the required tangent becomes as stated in the problem.

II. Solution by G. W. GREENWOOD, M.A., Professor of Mathematics, McKendree College, Lebanon, Ill., and J. SCHEFFER, Hagerstown, Md.

Denote the length of the tangent by l, and its equation by $y=mx+\sqrt{(a^2m^2+b^2)}$.

$\therefore l^2=(1+\frac{1}{m^2})(a^2m^2+b^2)$.

$l^2m^2=a^2m^4+b^2+a^2m^2+b^2m^2=(am^2-b)^2+m^2(a+b)^2$.

$\therefore l^2=(a+b)^2+\left(\frac{am^2-b}{m}\right)^2$.

Hence the minimum value of l is $a+b$.

Also solved by M. E. Graber, and W. L. Tryon.

197. Proposed by R. D. CARMICHAEL, Hartselle, Ala.

$$\int_0^\infty \frac{\sin mx \cos nx}{x}dx.$$

Solution by G. W. GREENWOOD, M. A., Lebanon, Ill.; M. E. GRABER, Tiffin, Ohio, and the PROPOSER.

The required integral may be written

$$\tfrac{1}{2}\int_0^\infty \left[\frac{\sin(m+n)x}{x}+\frac{\sin(m-n)x}{x}\right]dx,$$

and it therefore reduces to problem No. 186, [January, 1905, page 22]. If $m+n$ and $m-n$ are both positive, the result is $\frac{1}{2}\pi$. If both negative, $-\frac{1}{2}\pi$. If of opposite sign, 0.

Also solved by S. A. Corey.

189. Proposed by M. E. GRABER, A. M., Heidelburg University, Tiffin, O.

Show that $e\int_1^\infty$, $e^2\int_2^\infty$, ...$e^n\int_n^\infty$, ... are integers divisible by $(p+1)!$, when the expression under the integral is $z^p\Big[(z-1)\ldots\ldots(z-n)\Big]^{p+1}e^{-z}\,dz$.